

Linear inequality

Topic -1

Inequality and Linear Inequalities in one variable

Inequality

Two real numbers or two algebraic expressions related by the symbols $>$, $<$, $\leq$ or $\geq$ form an inequality or inequation.

In other words, a statement involving the symbol $>$, $<$, $\leq$ or $\geq$ is called inequality or inequation.

Here, the symbols $<$ (less than), $>$ (greater than), $\leq$ (less than or equal to) and $\geq$ (greater than or equal to) are known as symbol of inequalities.

e.g. $5 < 7$, $x \leq 2$, $x + y \geq 11$

Types of Inequality

i) Numerical inequality - An inequality which does not involve any variable is called a numerical inequality.

e.g. $4 > 2$, $8 < 21$

ii) Literal inequality - An inequality which have variable is called literal inequality.

e.g. $x < 7$, $y \geq 11$, $2x - y \leq 4$

iii) Strict inequality An inequality which have only $<$ or $>$ is called strict inequality.

e.g $3x + y < 0$, $x > \neq$

iv) slack inequality An inequality which have only $\geq$ or $\leq$ is called slack inequality

LINEAR INEQUALITY

An inequality said to be linear, if the variable(s) occurs in first degree only and there is no involving the product of the variable.

e.g $ax + b \leq 0$, $ax + by + c > 0$, $ax \leq 4$

Linear Inequality in One Variable

A linear inequality which has only one variable, is called linear inequality in one variable.

e.g $ax + b < 0$, where $a \neq 0$

Note:

An inequality in one variable, in which degree of variable is 2, is called quadratic inequality in one variable

e.g $ax^2 + bx + c \geq 0$ $a \neq 0$; $3x^2 + 2x + 1 \leq 0$

Linear Inequality in Two Variable

A linear inequality which have only two variable, is called linear inequality in two variable

e.g $3x + 11y \leq 0$, $4x + 3y > 0$

CONCEPT OF INTERVALS ON A NUMBER LINE

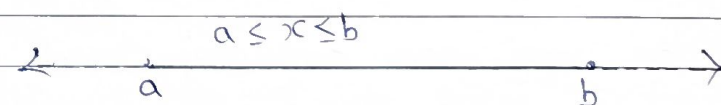
On number line or real line, various types of infinite subsets known as intervals, are defined below.

Closed interval

If a and b are real numbers, such that $a < b$, then the set of all real numbers x , such that $a \leq x \leq b$ is called interval and is denoted by $[a, b]$

$$\therefore [a, b] = \{x : a \leq x \leq b, x \in \mathbb{R}\}$$

On the number line $[a, b]$ may be represented as follows



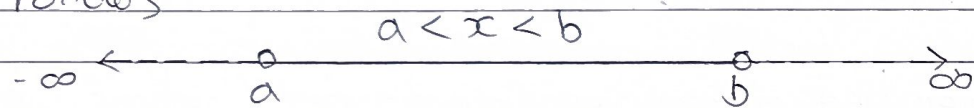
Here, end points of the interval i.e. a and b are included in the interval. So, on number line, draw filled circle ($\bullet$) at a and b

OPEN Interval

If a and b are real numbers such that $a < b$, then the set of all real number x , such that $a < x < b$, is called open interval and it denoted by (a, b) or $]a, b[$.

$$\therefore (a, b) = \{x : a < x < b, x \in \mathbb{R}\}$$

On the number line, (a, b) may be represented as follows



Here, end point of the interval i.e. a and b are not included in the interval.

Semi-Open or Semi-closed interval.

If a and b are real numbers, such that $a < b$ then

$$[a, b] = \{x : a \leq x \leq b, x \in \mathbb{R}\}$$

$$\text{and } [a, b) = \{x : a \leq x < b, x \in \mathbb{R}\}$$

are known as semi-open or semi-closed interval

Addition or Subtraction

Same number may be added (or subtracted) both sides of an inequality i.e. if $a > b$, c ,

$$a + c > b + c, a - c > b - c$$

$$\text{i.e. (i) } 10 > 5 \rightarrow 10 + 7 > 5 + 7 \text{ (adding 7 on both sides)}$$

$$\rightarrow 17 > 12, \text{ which is true}$$

$$\text{ii) } -8 > -13 \rightarrow -8 - 2 > -13 - 2 \text{ (subtracting 2 from both sides)}$$

$$\rightarrow -10 > -15, \text{ which is true.}$$

Multiplication or Division

If both sides of an inequality are multiplied or divided by the same positive number, then the sign of inequality remain the same. But when both sides are multiplied or divided by the same negative number, then the sign of inequality is reversed.

Let a and b and c be three real number, such that $a > b$ and $c \neq 0$

$$\text{i) If } c > 0, \text{ then } \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc$$

$$\text{ii) If } a > b \text{ and } c < 0 \text{ then } \frac{a}{c} < \frac{b}{c}$$

$$\text{eg a) } 6 > 4 \rightarrow 6 \times 2 > 4 \times 2 \text{ (multiply)} \\ \rightarrow 12 > 8, \text{ which is true.}$$

Solution set

The set of all solutions of an inequality is called the solution set of the inequality.

$$b \\ -32 < -24$$

$$\Rightarrow \frac{-32}{8} > \frac{-24}{8} \\ \text{(divided)}$$

$$\rightarrow 4 > 3, \text{ which is true}$$

Algebraic Solution of Linear Inequality in one Variable

Any solution of an linear inequality in one variable is a value of the variable which make it a true statement.

e.g. $x=1$ is the solution of the linear inequality $4x+7 > 0$

Topic 2

System of Inequalities in one variable and Their Solutions

Two or more inequalities taken together comprise a system of inequalities and the solution of the system of inequalities are the solution common to all the inequalities comprising the system

e.g. $x=10$ is the solution of the system of inequalities

$$4x+3 \leq 91 \text{ and } 2x \geq x+8$$

Solution of system of linear inequality in one variable.

We know that, the solution set of a linear inequality in one variable is the set of all points on the number line satisfying the given inequality.

Therefore the solution set of a system of linear inequality in one variable is defined as the intersection of the solution set of the linear inequalities in the system.

e.g. If the solution set of linear inequalities in the system are $(-\infty, 5]$ and $[5, \infty)$ then the solution of the system of linear inequalities in one variable is 5 only. Because, if we represent the solution sets on the number line, we see that the value which are common to both is 5 only



The process of finding solution of system of linear inequality of different types are given below

Type 1

WHEN TWO SEPARATE LINEAR INEQUALITIES ARE Given.

If the given system of inequalities comprise by two separate linear inequalities, then to solve these we use the following working steps.

Step I = Solve each inequality separately and obtain their solution sets

Step II = Represent the solution sets on a number line and then find the values of the variable which are common to them.

Type 2

When Inequalities of the form

$$a \leq \frac{cx+d}{e} \leq b, \text{ where } a, b, c, d \in \mathbb{R}$$